The Newton power flow method for unbalanced three-phase distribution networks

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***Abstract*—A general framework is given for applying the Newton-Raphson method to solve three-phase power flow problems, using power and current-mismatch functions in polar, Cartesian coordinates and complex form. These two mismatch functions and three coordinates, result in six versions of the Newton-Raphson method. A three-phase power flow formulation of all versions is described for PQ-buses and PV-buses. This framework enables us to compare all variants theoretically. Furthermore, the convergence behavior is investigated by numerical experiments. Newly developed/improved versions of the Newton power flow method are compared with the Backward-Forward Sweep based algorithm. We conclude that the polar current-mismatch and Cartesian current-mismatch versions that are developed in this paper, performed the best result for both balanced and unbalanced distribution networks.**

I. Introduction

Due to the modernization of the existing grid, a large number of new grid elements and functions are being integrated into the grid. In the smart grid, most of the new grid elements are directly connected to the distribution network which requires a new operation and maintenance. In addition, the distribution network has some special features that the transmission network does not have:

- Radial or weakly meshed structure
- High resistance and reactance ratio R/X
- Multi-phase power flow and unbalanced loads
- Distributed generations

For an efficient operation and planning of the power system, it is essential to know the system steady state conditions for various load demands. A power flow computation that determines the steady state behavior of the network is one of the most important tools for grid operators. The solution of the power flow computation can be used to assess whether the power system can function properly for the given generation and consumption. The conventional power flow solution techniques on a transmission network are Gauss-Seidel (G-S), Newton power flow (N-R) and Fast Decoupled Load Flow (FDLF) [1]–[3], which are widely used for power system operation, control and planning. In practice, the Newton power flow method is preferred in terms of better convergence and robustness properties. Many methods have been developed on distribution power flow analysis and generally they can be divided into two main categories as:

- Modification of conventional power flow methods
- Backward-Forward Sweep (BFS)-based algorithms

Excellent reviews on distribution load flow solution methods can be found in [4]–[7].

A reliable distribution power flow solution method will be required to solve three-phase power flow for both meshed and radial unbalanced distribution network integrated with distributed and active resources (i.e. DGs, storages devices and electric vehicles etc) [5], [6]. Therefore, in this paper we want to obtain a fast and robust power flow solution technique that can solve most complex distribution networks with any topology. We want to achieve this goal by doing the comparison study for three-phase power flow on weakly-meshed unbalanced distribution network with DGs and active resources.

II. Three-phase power flow problem

The power flow, or load flow, problem is the problem of computing the voltage *magnitude* $|V_i|$ and *angle* δ_i in each bus of a power system where the power generation and consumption are given. The mathematical equations for the power flow problem are given by:

$$S_i^p = V_i^p (I_i^p)^* = V_i^p \sum_{k=1}^{N} \sum_{q=a,b,c} (Y_{ik}^{pq})^* (V_k^q)^* \qquad (1)$$

where N is the total number of buses in the network, S_i^p injected complex power, V_i^p is the voltage and I_i^p is the injected current at bus i for a given phase p.

A. The power mismatch function

The power flow problem (1) is formulated as the power mismatch function $F(\vec{x})$ as follows:

$$F_i(\vec{x}) = \Delta S_i^p = (S_i^{sp})^p - V_i^p \sum_{k=1}^{N} \sum_{q=a,b,c} (Y_{ik}^{pq})^* (V_k^q)^*$$

where $(S_i^{sp})^p = (P_i^{sp})^p + \imath (Q_i^{sp})^p$ is the specified complex power at bus i for a given phase p.

B. The current mismatch function

The power flow problem (1) is formulated as the current mismatch function $F(\vec{x})$ as follows:

$$F_i(\vec{x}) = \Delta I_i^p = \left(\frac{(S_i^{sp})^p}{V_i^p}\right)^* - \sum_{k=1}^{N} \sum_{q=a,b,c} Y_{ik}^{pq} V_k^q.$$

III. Newton power flow solution method

Depending on problem formulations (power or current mismatch) and coordinates (polar, Cartesian and complex form), the Newton-Raphson method can be applied in six different ways as a solution method for power flow problems. The Newton based power flow methods use the Newton-Raphson (NR) method that is applied to solve a nonlinear system of equations $F(\vec{x}) = 0$. The linearized problem is constructed as the Jacobian matrix equation

$$-J(\vec{x})\Delta\vec{x} = F(\vec{x}) \tag{2}$$

where $J(\vec{x})$ is the square Jacobian matrix and $\Delta\vec{x}$ is the correction vector. The Jacobian matrix is obtained by $J_{ik} = \frac{\partial F_i(\vec{x})}{\partial x_k}$ and is highly sparse in power flow applications [2], [8]. Traditionally, a direct solver is used to solve the Jacobian matrix equation. Convergence of the method is mostly measured in the residual norm $||F(\vec{x}^h)||_{infty}$ of the mismatch function at each iteration. The Newton power flow method has a quadratic convergence when iterates are close enough to the solution.

IV. Numerical experiment

The newly developed/improved versions of the Newton power flow method (NR-p-car, NR-p-com, NR-c-pol, NR-c-car and NR-c-com) are compared to the existing version (NR-p-pol [2]) and Backward-Forward Sweep based algorithm (BFS [9]). Two balanced distribution networks (33-bus [10] and 69-bus [11]) and two unbalanced distribution networks (13-bus [12] and 37-bus [12]) are used to test the convergence ability and scalability of all the versions of the Newton power flow solution method. All methods are implemented in Matlab and the relative convergence tolerance is equal to 10^{-5}. The maximum number of iteration is equal 10. All experiments are performed on an Intel computer with four cores i5-4690 3.5GHz CPU and 64Gb memory, running a Debian 64-bit Linux 8.7 distribution.

A. Balanced single-phase distribution networks

Methods	Test cases					
	DCase33			DCase69		
	iter	time	$\|\|F(\vec{x})\|\|$	iter	time	$\|\|F(\vec{x})\|\|$
NR-p-pol	3	0.0096	7.467e-09	3	0.0089	1.042e-08
NR-p-car	3	0.0061	1.043e-09	3	0.0066	8.177e-09
NR-p-com	3	0.0050	3.133e-06	3	0.0050	2.838e-06
NR-c-pol	2	0.0081	4.067e-06	2	0.0090	7.645e-06
NR-c-car	2	0.0070	4.004e-06	3	0.0082	1.948e-11
NR-c-com	3	0.0054	9.938e-06	5	0.0068	9.742e-06
BFS	4	0.0163	2.300e-06	5	0.0104	9.421e-07

TABLE I: Balanced distribution networks

B. Unbalanced three-phase distribution networks

Methods	Test cases					
	DCase13			DCase37		
	iter	time	$\|\|F(\vec{x})\|\|$	iter	time	$\|\|F(\vec{x})\|\|$
NR-p-pol	2	0.0087	7.649e-06	2	0.0092	1.837e-08
NR-p-car	3	0.0062	2.743e-11	2	0.0078	1.938e-08
NR-p-com	4	0.0055	7.153e-07	2	0.0051	1.705e-07
NR-c-pol	2	0.0081	2.822e-06	2	0.0080	3.463e-09
NR-c-car	2	0.0075	3.069e-06	2	0.0072	3.988e-09
NR-c-com	4	0.0056	2.630e-06	2	0.0052	6.178e-07

TABLE II: Unbalanced distribution networks

V. Conclusion

From table I and II, we see that the Newton power flow versions using the current-mismatch functions have better convergence than versions using the power-mismatch functions regardless of the choice of the coordinates. Although the complex power-mismatch and the complex current-mismatch versions have the same number of iterations to converge, these versions have linear convergence whereas other versions have quadratic convergence. Thus, the complex versions are least preferable to the Newton power flow versions. In addition, the Cartesian current-mismatch version has an advantage in the calculation of the Jacobian matrix because its off-diagonal elements are constant and equal to the terms of the nodal admittance matrix. Moreover, depending on the properties of given network, one version can work better than the other versions. Therefore, it is crucial to study which version is more suitable for what kind of power network.

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