Assessing safe operating regions in power grids under uncertainty

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***Abstract*—We consider a static model of a power grid using a DC approximation. A line overload occurs when a current exceeds a critical value. We investigate the probability of line overloads and derive expressions for safe capacity regions which can be incorporated in classical planning methods like Optimal Power Flow (OPF), taking uncertainty into account.**

I. Introduction

This abstract is strongly based on the conference paper [1], and is motivated by the need of keeping electrical power grids reliable while retaining existing planning methods, like optimal power flow (OPF). The rise of intermittent renewable generation is making this important, as a planning that accounts for worst-case behavior may lead to overly conservative solutions. A more realistic paradigm is to make a planning admissible when the probability that line power flows exceed a threshold is sufficiently small. This has motivated several recent works that attempt to evaluate line overload probabilities using rare event simulation techniques [2]–[4], as well as large deviations techniques [5]. Simulation techniques can lead to accurate estimates, but may be too time-consuming to use as subroutine within an optimization package that has to determine a planning that is operational during the next 5 to 15 minutes, such as OPF. Recent papers studying chance-constrained versions of OPF problems include [6], [7].

Our present research focuses on large deviations techniques, and in addition presents approximations for line overload probabilities that are *guaranteed to be conservative*. That is, our methodology ensures that reliability constraints on active power flow are actually met. In addition, these new approximations are explicit enough to be used for optimization purposes on short time scales. In particular, we develop two such approximations in Section III. Both bounds lead to an approximation of the capacity region that is conservative, convex and polyhedral, making our results compatible with existing planning methods like OPF [6], [7].

II. Model description

We model the power grid network as a connected graph $G = G(V, E)$, where V denotes the set of *buses* and E the set of directed edges modeling the *transmission lines*. $n = |V|$ is the number of buses and $m = |E|$ is the number of lines. $(i, j) \in E$ denotes the transmission line between buses i and j with *susceptance* $\beta_{i,j} = \beta_{j,i} > 0$. If there is no transmission line between i and j we set $\beta_{i,j} = \beta_{j,i} = 0$.

As in [8], [9], the network structure and susceptances are encoded in the *weighted Laplacian matrix* L. Denote by $p \in \mathbb{R}^n$ the vector of (real) power injections and by $\widetilde{f} \in \mathbb{R}^m$ the vector of (real) power flows over the lines. We will use the convention that $p_i \geq 0$ ($p_i < 0$) means that power is generated (consumed, respectively) at bus i.

We make use of the *DC approximation*, which is commonly used in transmission system analysis [10]–[13]. Under this approximation, line power flows $\widetilde{f}$ can be written as a linear transformation of the power injections p. Transmission lines can fail due to overload. We say that a *line overload* occurs in transmission line ℓ if $|\widetilde{f}_\ell| > M_\ell$, where M_ℓ is the *line capacity*. If this happens, the line may trip, causing a global redistribution of the line power flows which could trigger cascading failures and blackouts. It is convenient to look at the *normalized line power flow* vector $f \in \mathbb{R}^m$, defined component-wise as $f_\ell := \widetilde{f}_\ell / M_\ell$ for every $\ell = 1, \dots, m$ and in matrix form as $f = Cp$, where C depends on the network topology, susceptances, and line capacities, for details see [1].

We assume that bus n is a *slack bus*, and that the vector of the first $n-1$ power injections $(p_1, \dots, p_{n-1})$ follows a multivariate Gaussian distribution, with expected value $\mu \in \mathbb{R}^{n-1}$ and covariance matrix $\Sigma \in \mathbb{R}^{(n-1)\times(n-1)}$. This formulation allows us to model *deterministic power injections* as well, by means of choosing the corresponding variances and covariances equal to zero. Following the construction in [1], we can write for specific matrices V, W,

$$f = VX + W\mu,$$

where X is a $(n-1)$-dimensional standard multivariate Gaussian. Note in particular that $f_\ell \sim \mathcal{N}(\nu_\ell, \sigma_\ell^2)$, where $\nu = W\mu$ and

$$\sigma_\ell^2 := \sum_{j=1}^{n} V_{l,j}^2.$$

III. Main results

Our goal is to understand how the probability of an overload violation depends on the parameters of the systems and characterize which average power injection vector μ will make such a probability smaller than a desired target.

In view of the definition of line overload given in Section II, we define the *line overload event* $\mathcal{L}$ as

$$\mathcal{L} = \left\{ \max_\ell |f_\ell| \geq 1 \right\}.$$

We aim to characterize for a *given* covariance matrix Σ the average power injection vectors μ that make line overloads a *rare event*, say $\mathbb{P}_\mu(\mathcal{L}) \le q$ for some very small threshold $q \in (0,1)$ to be set by the network operator (think of $q = 10^{-3}$ or $q = 10^{-4}$). In other words, given $q \in (0,1)$, we aim to determine the region $\mathcal{R}_q^{\text{true}} \subset \mathbb{R}^{n-1}$ defined by

$$\mathcal{R}_q^{\text{true}} := \{\mu \in \mathbb{R}^{n-1} : \mathbb{P}_\mu(\mathcal{L}) \le q\}.$$

For every given $\mu \in \mathbb{R}^{n-1}$, calculating exactly the probability $\mathbb{P}_\mu(\mathcal{L})$ means solving a high-dimensional integral that is also unavoidably error-prone, since the integrand becomes extremely small quickly (containing a multivariate Gaussian density). Hence, finding the region $\mathcal{R}_q^{\text{true}}$ exactly is a very computationally expensive and error-prone task.

Two of our main results are the following two approximations of $\mathcal{R}_q^{\text{true}}$ (set $a := (\sqrt{2\log(1/q)} + \sqrt{2\log(2m)})$):

$$\mathcal{R}_q^{\text{up}} = \Big\{\mu : |\nu_\ell| \le 1 - a \max_{\ell'} \sigma_{\ell'} \ \ \forall \ell\Big\},$$
$$\mathcal{R}_q^{\text{ld}} = \Big\{\mu : |\nu_\ell| \le 1 - \sigma_\ell \sqrt{\log(1/q)} \ \ \forall \ell\Big\}.$$

The first approximation is based on an analytic upper bound for $\mathbb{P}_\mu(\mathcal{L})$ which is derived using the Borell inequality, and by bounding the expected value of $\max_\ell |f_\ell|$. It is a conservative approximation, i.e. all $\mu \in \mathcal{R}_q^{\text{up}}$ are guaranteed to have a overload probability smaller than q. The region $\mathcal{R}_q^{\text{ld}}$ is derived using the theory of large deviations, and is accurate when $\max_\ell \sigma_\ell$ is small. We remark that in the same small noise regime (i.e. $\max_\ell \sigma_\ell \ll 1$) the approximation of $\mathcal{R}_q^{\text{up}}$ is still meaningful for large power grids as a grows only as $\sqrt{2\log(2m)}$ with the number of lines m. For instance if $q = 10^{-4}$, then $a \approx 8.19$ for $m = 10^3$ and $a \approx 8.74$ for $m = 10^4$. The smaller region $\mathcal{R}_q^{\text{up}}$, although more conservative, is expressed in closed-form and, moreover, its dependency on the parameters ν, σ and m is made explicit. In particular, the maximum standard deviation of the power flows, i.e. $\max_\ell \sigma_\ell$ plays a big role in defining the capacity regions: indeed to larger values of $\max_\ell \sigma_\ell$ correspond smaller regions, which is intuitive since a bigger variance results in a higher probability of overload.

In [1] the additional regions $\mathcal{R}_q^{\text{c.i.}}$ and $\mathcal{R}_q^{\star}$ are introduced, which lie between the two regions $\mathcal{R}_q^{\text{up}}$ and $\mathcal{R}_q^{\text{ld}}$. In particular, $\mathcal{R}_q^{\star}$ is less conservative than $\mathcal{R}_q^{\text{up}}$ and can be computed very efficiently, even if it cannot be expressed in closed-form. All the regions $\mathcal{R}_q^{\text{up}}$, $\mathcal{R}_q^{\star}$ and $\mathcal{R}_q^{\text{ld}}$ seem sufficiently explicit to be used as probabilistic constraints into chance-constrained versions of OPF problems, as studied in [6], [7].

IV. Numerical example

To illustrate how the developed regions compare to $\mathcal{R}_q^{\text{true}}$, we consider a very simple network with a circuit topology, consisting of 3 buses, all connected with each other by 3 identical lines of unit susceptance and capacity $M = 5$. We take the power injections in the 2 non-slack nodes to be independent Gaussian random variables with variance $\epsilon = 0.5$, which corresponds to take $\Sigma = \epsilon I_2$. The corresponding safe capacity regions for $q = 10^{-3}$, obtained by varying the average power injections (μ_1, μ_2), are plotted in Figure 1.

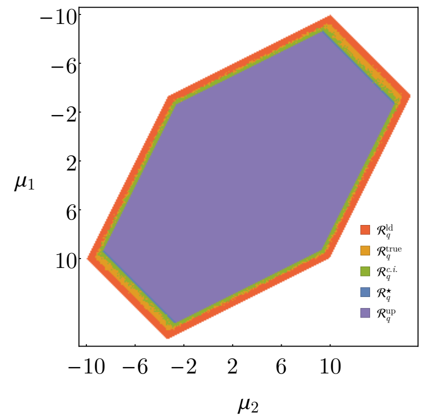

Figure 1: Capacity regions comparison for a 3-bus cycle network

V. Concluding remarks

Probabilistic techniques, in particular powerful upper bounds for Gaussian random vectors as well as large deviations techniques can be applied to generate explicit approximations for overload probabilities and corresponding safe capacity regions. The resulting regions are polyhedral, and can be characterized in such a way that they can be incorporated in optimization routines, such as OPF.

In our presentation we will show how our methodology can be extended to analyze the likelihood of multiple overloads, showing in particular that subsequent overloads may not be of nearest-neighbor type.

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